Behavior of Particles in a Liquid-Solids Fluidized Bed

The mechanics of particle motion in a fluidized bed were studied by focusing on the microscopic behavior of the particles. The local displacements of nylon particles fluidized by liquid were measured by the micro-capacitance method, and the resultant time series was analyzed by determining its auto-correlation function and the corresponding power spectrum. This has given rise to a stochastic or statistical model of particle displacements in the fluidized bed. This model visualizes the particle motion in a fluidized bed to consist of the random movement, generating irregular signals, and the linear movement, generating wave-like signals.

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SCOPE

The behavior of particles in liquid-solids fluidized beds was studied by several investigators. For example, Carlos and Richardson (1968) measured the particle velocity distribution by a photographic method. Bailie (1965) determined the motion of an individual particle by means of a γ-ray detector. Slis et al. (1958) and Fan et al. (1963) determined the behavior of particles during the unsteady bed-expansion by means of response techniques. However, these investigations were mainly empirical and essentially macroscopic in nature.

Recently, Yutani and Ototake (1980) analyzed the motion of particles in a liquid-solids fluidized bed by a statistical method. To enhance the theoretical basis of their approach, additional microscopic measurements need to be made, and a mechanistic model for the particle motion needs to be developed.

In this study, local displacements of nylon particles fluidized by a fluidizing medium were measured by the micro-capacitance method. The resultant time series were analyzed by determining their auto-correlation function and power spectrum. An auto-correlation function gives rise to the variance of particle displacements and the relaxation time of particle movement in the field; it can provide useful information for estimating the local particle diffusivity which is of practical importance in understanding mechanism and in determining the mass or heat transfer between the fluid and particles in a fluidized bed. The power spectrum yields information related to the local velocity distribution of particles.

CONCLUSIONS AND SIGNIFICANCE

A new technique, which utilizes a micro-capacitance probe, has been developed to measure local fluctuations of particle motion in a liquid-solids fluidized bed. The results indicate that the recorded signals are composed of a sine wave component and a stochastic component. A model has been proposed by taking these two components into account, and the auto-correlation function of the model has been fitted to experimental data by adjusting the frequency of the sine wave, f_o, and the intensity of transition of particles, λ. In addition, f_o/λ has been found to be independent of the void fraction of the fluidized bed, ϵ, but dependent on the viscosity of the fluidizing medium, μ. The power spectral density function derived from the model has been compared with that from the data. Although a liquid-solids fluidized bed is used in this study, there is no reason to suspect that the methodology developed here can not be applied to a gas-solids fluidized bed. In fact, it is easier to use the micro-capacitance probe in the latter than in the former because the probe need not be insulated in the gas-solids bed.

THEORETICAL

Mechanism of Particle Motion

Experimental observations (e.g., Houghton, 1966; Carlos and Richardson, 1968) of the behavior of particles in liquid-solids fluidized beds have revealed that flow of the fluidizing medium induces not only convective flow of groups of particles but also random motion of the individual particles. A mechanism is proposed here for the fluctuations of particle motion, i.e., the microscopic change in the movement of particles, in a macroscopically stationary fluidized bed.

Suppose that a micro-capacitance probe is located at a given position in the fluidized bed under examination. Then, the voltage across the probe is affected by the occurrence of passage of a particle in the field around the probe, Figure 1. This figure depicts the change in the electric voltage experienced by the probe. This change is affected by the characteristics of the field of the probe and the particle motion. A particle passing through the field gives rise to a segment of wave-like signals. Furthermore, these wave-like signals are distorted by the randomness of particle movements into and out of the field.

Assume that the recorded signals consist of two components. The first is the periodic or sine wave component, $Y(t)$, measured from its mean value. This can be expressed as

$$Y(t) = A \sin (2\pi f_o t + \theta) \tag{1}$$

where

A = amplitude
f_o = frequency in cycles per unit time
θ = initial phase angle with respect to the time origin in radians

Since a particle enters the field of a micro-capacitance probe in a random manner, θ is a random variable; it is assumed that θ is uniformly distributed with a probability density function, $p(\theta)$,

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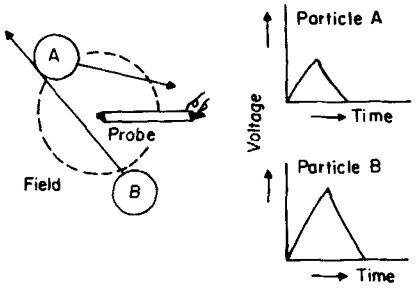

Figure 1. Schematic relationship between the variation of voltage and the passage of a particle in the field of the probe.

over $(0,2\pi)$. The second component is represented by a stochastic process describing the random occurrence of passage of a particle through the field, for which we define

$$Z(t) = \begin{cases} 1, \text{ if a particle is in the field} \\ 0, \text{ otherwise} \end{cases} \quad (2)$$

This stochastic process is completely defined under the following assumptions (e.g., Chatfield, 1975):

$$Pr[Z(t + \Delta t) = 1 | Z(t) = 0] = \alpha \Delta t + o(\Delta t) \quad (3)$$

and

$$Pr[Z(t + \Delta t) = 0 | Z(t) = 1] = \beta \Delta t + o(\Delta t) \quad (4)$$

where α is the intensity of transition of a particle from the outside of the field to the inside, and β from the inside to the outside. The expression in the left-hand side of Eq. 3 or 4, namely, $Pr[E_1|E_2]$, denotes the conditional probability of event E_1 given event E_2. Let

$$p_A(t) = Pr[Z(t) = 1] \quad (5)$$

Then, it can be shown that (see the Appendix)

$$p_A(t) = \frac{\alpha}{(\alpha + \beta)} + \left[p_A(0) - \frac{\alpha}{(\alpha + \beta)}\right] e^{-(\alpha+\beta)t} \quad (6)$$

From the characteristics of the capacitance probe employed in the present work, which will be delineated in the succeeding section, we see that the wave-like component, $Y(t)$, and the stochastic component, $Z(t)$, of the recorded signals, $X(t)$, are independent of each other but occur simultaneously. Thus, we can write

$$X(t) = Y(t)Z(t) \quad (7)$$

Auto-Correlation Function and Power Spectrum

The auto-correlation function of random data describes the general dependence of the datum at one moment on that at another moment (e.g., Bendat, 1958; Chatfield, 1975).

Consider a sampled time history record of $X(t)$. An estimation for the auto-correlation between the values of $X(t)$ at times t and $(t + \tau)$ is obtained by taking the product of the two values and averaging over the observation time, T. The resultant average product will approach the exact auto-correlation function of $X(t)$ as T approaches infinity, i.e.,

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T X(t)X(t + \tau)dt \quad (8)$$

Based on the assumption that the "ergodic hypothesis" is valid for the system under consideration, the auto-correlation function is equal to the corresponding ensemble averaged value, i.e.,

$$R_{xx}(\tau) = E[X(t)X(t + \tau)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1x_2p(x_1,x_2;\tau)dx_1dx_2 \quad (9)$$

where $p(x_1,x_2;\tau)$ is the joint probability density function of $X(t)$ and $X(t + \tau)$.

Applying the definition of auto-correlation function, as given by Eq. 9, to the periodic component of the recorded signals, $Y(t)$ given by Eq. 1, we have

$$R_{yy}(\tau) = \int_0^{2\pi} A \sin(2\pi f_o t + \theta) A \sin[2\pi f_o(t + \tau) + \theta] p(\theta)d\theta = \frac{A^2}{2}\cos 2\pi f_o \tau \quad (10)$$

Since $Z(t)$ is assumed to be a stationary random process, the auto-correlation of $Z(t)$ can be derived as (see the Appendix)

$$R_{zz}(\tau) = \gamma e^{-\lambda\tau} \quad (11)$$

Since $Y(t)$ and $Z(t)$ are independent of each other, Eq. 7 indicates that the auto-correlation function of $X(t)$ can be written as (e.g., Bendat, 1958)

$$R_{xx}(\tau) = R_{yy}(\tau)R_{zz}(\tau) \quad (12)$$

Substitution of Eqs. 10 and 11 into this expression gives

$$R_{xx}(\tau) = \frac{\gamma A^2}{2} e^{-\lambda\tau} \cos 2\pi f_o \tau \quad (13)$$

The power spectral density function of $X(t)$, $G_{xx}(\omega)$, is defined as (e.g., Bendat, 1958)

$$G_{xx}(\omega) = \frac{2}{\pi}\int_0^{\infty} R_{xx}(\tau)\cos \omega\tau d\tau \quad (14)$$

Substitution of Eq. 13 into this expression gives

$$G_{xx}(\omega) = \frac{\gamma\lambda A^2}{\pi}\left[\frac{\omega^2 + (\lambda^2 + c^2)}{\omega^4 + 2(\lambda^2 - c^2)\omega^2 + (\lambda^2 + c^2)^2}\right] \quad (15)$$

where

$$c = 2\pi f_o \quad (16)$$

At the stationary point in $G_{xx}(\omega)$, the condition

$$G'_{xx}(\omega) = \frac{\gamma\lambda A^2}{\pi}\frac{(-2\omega)[\omega^4 + 2(\lambda^2 + c^2)\omega^2 + (\lambda^4 - 2\lambda^2c^2 - 3c^4)]}{[\omega^4 + 2(\lambda^2 - c^2)\omega^2 + (\lambda^2 + c^2)^2]^2} = 0 \quad (17)$$

holds. The value of ω satisfying this condition is

$$\omega = (\lambda^2 + c^2)^{1/4}[2c - (\lambda^2 + c^2)^{1/2}]^{1/2} = \omega_1 \quad (18)$$

We find that the value of the second derivative at this point is negative, i.e.,

$$G''_{xx}(\omega)|_{\omega=\omega_1} < 0$$

Thus, a single maximum of $G_{xx}(\omega)$ occurs at ω_1, and the corresponding value of this maximum is

$$G_{xx}(\omega_1) = \frac{\gamma\lambda A^2}{4c}\frac{1}{[(\lambda^2 + c^2)^{1/2} - c]} \quad (19)$$

EXPERIMENTAL

Apparatus

The experimental apparatus and flow system employed are illustrated schematically in Figure 2. This system involved a head tank, two orifice flow meters, flow-regulation valves, and a fluidized bed proper, in which a highly sensitive micro-capacitance probe was located for measuring fluctuations of the particle movement. A stainless-steel screen with an opening of 300-mesh was utilized as a distributor, below which a packed bed of glass beads with an average diameter of 0.647 mm (24–28 mesh) served as the calming section. The fluidized bed proper was made of acrylic resin with an inner diameter of 50 mm and a height of 700 mm.

Procedure

A given quantity of nylon particles with an average diameter of 6.5 mm was poured into the bed and fluidized by maintaining the superficial velocity of the fluidizing medium at U. The fluctuations of particle motion

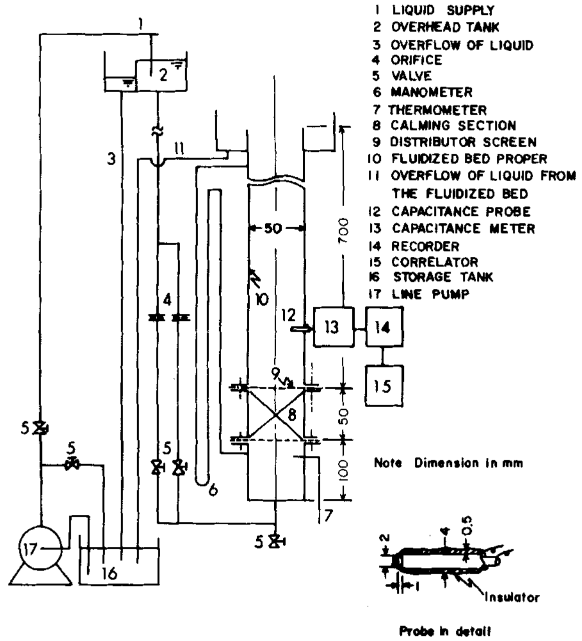

Figure 2. Schematic of the experimental set-up.

around the probe were recorded by means of the micro-capacitance probe. Experiments were repeated by varyng the superficial velocity of the fluid and the viscosity of the fluid.

Calibration of the Probe

The probe was calibrated according to the procedure described below.

(1) A single particle was fixed at an arbitrary location in the field of the probe.
(2) The signal from the probe was recorded.
(3) The first two steps were repeated by changing the location of the particle.

The relationship between the magnitude of signals and the distance from the center of the probe to that of the particle was determined from the result obtained, as illustrated in Figure 3. Notice that the relationship is essentially linear.

RESULTS AND DISCUSSION

Auto-Correlation Function of Recorded Signals

Typical signals recorded are traced in Figure 4, with the superficial velocity of the fluidizing fluid as the parameter. It appears

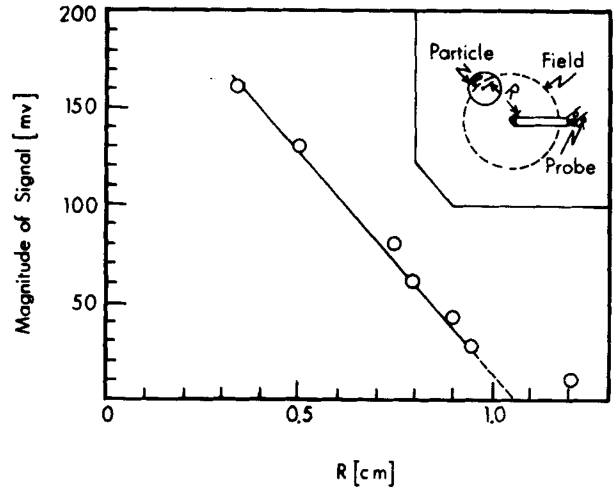

Figure 3. Calibration curve for the micro-capacitance probe.

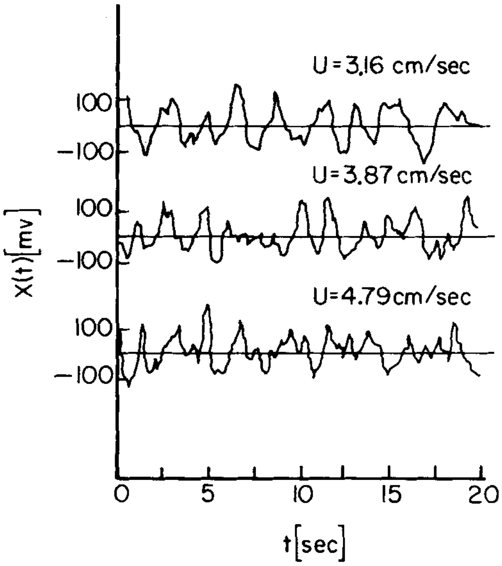

Figure 4. Recorded signals at various superficial velocities of the fluidizing medium.

that each tracing of the recorded signals at a given superficial velocity contains wave-like signals with various frequencies. In other words, each tracing of the recorded signals is composed of a random or stochastic component and a wave-like or sine component.

The auto-correlation function expresses the extent of the correlation between a signal at an arbitrary moment, t, and that at time $(t + \tau)$. Three typical auto-correlation functions obtained experimentally are illustrated in Figures 5, 6 and 7. Characteristics of the periodic and stochastic components can be evaluated by analyzing the auto-correlation functions.

Comparison of the Model with the Experimental Data

The auto-correlation function based on the model, Eq. 13, has been fitted to that obtained experimentally by manipulating the frequency, f_o, and the intensity, λ. This is illustrated in Figure 8. The relationship between the frequency, f_o, and the experimentally determined average void fraction of the bed, ϵ, is given in Figure 9, where the viscosity of the fluidizing medium, μ, is the parameter.

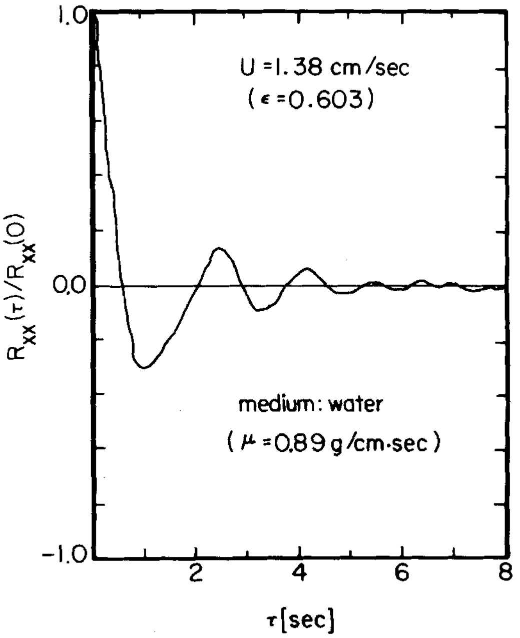

Figure 5. Typical auto-correlation function in the range of void fraction from 0.6 to 0.7.

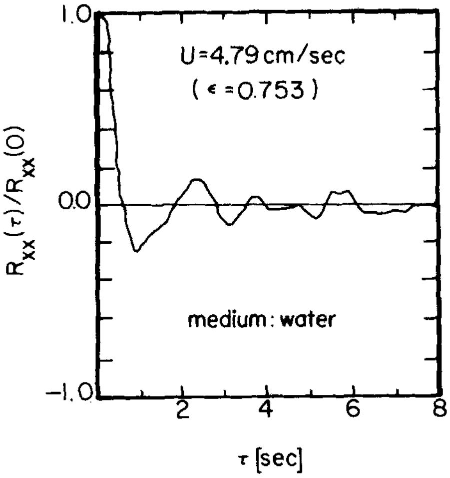

Figure 6. Typical auto-correlation function in the range of void fraction from 0.7 to 0.8.

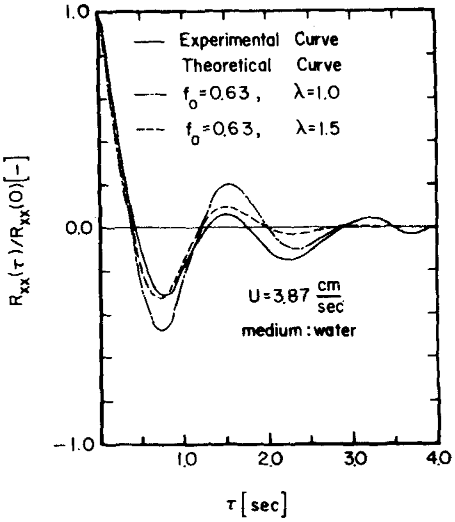

Figure 8. Illustration of fitting of the auto-correlation function based on the model to the experimentally determined function.

Note that in the range of the void fraction between 0.6 and 0.7, the periodicity of signals is distinct (Figure 5) and their frequencies, f_o, increase with an increase in the void fraction (Figure 9); on the other hand, in the range from 0.7 to 0.8, the periodicity is blurred (Figure 6), and the wave-like characteristics disappear gradually with an increase in the void fraction (Figure 9). If the void fraction is over 0.8, the wave-like signals reappear, and their frequencies, f_o, also increase with an increase in the void fraction (Figures 7 and 9). The frequency of particles passing through the field of the probe is closely related to the void fraction of the bed, ϵ. Apparently two counteracting effects are operating. ϵ increases when the velocity of the fluidizing medium is increased. Consequently, particles tend to move faster with less hinderance, thus encountering the probe more frequently. On the other hand, the density of particles around the probe is reduced as ϵ increases, causing a smaller number of particles to encounter the probe. These two effects dominate in different operating regimes.

The existence of a periodic component in turbulent flow of even a homogeneous fluid has been observed or proved (e.g., Gilmore, 1981). The fluidized bed is a heterogeneous mixture of solid particles and a fluid which is in motion; therefore, the existence of a highly chaotic or stochastic component is more than plausible due to the collision and relative motion among the solid particles and the interaction between the solid particles and fluid. Furthermore, the relationship between the intensity, λ, and the void fraction, ϵ, is similar to that between the frequency, f_o, and the void fraction, ϵ, as can be seen from comparing Figures 9 and 10. This implies that f_o/λ becomes essentially independent of the void fraction, ϵ; it is only a function of the viscosity of the fluidizing medium, μ. This gives rise to the following relationship:

$$\frac{f_o}{\lambda} = g(\mu) \tag{20}$$

This relationship is shown in Figure 11.

Equation 20 appears to suggest that movement of particles in the bed satisfies the relationship analogous to the "Einstein Relation" in the Brownian motion, which is expressed as (e.g., Kubo, 1957; Hori, 1977).

$$\frac{D}{\zeta} = f(T_m) \tag{21}$$

where D is the particle diffusivity in a suspended Brownian par-

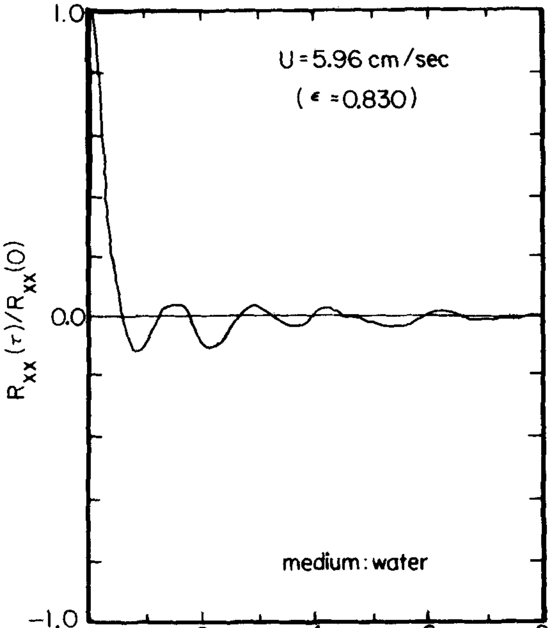

Figure 7. Illustration of fitting of the auto-correlation function based on the model to the experimentally determined function.

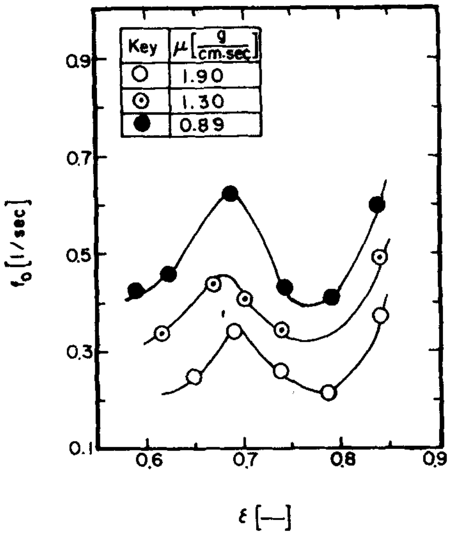

Figure 9. Relationship between the frequency of wave-like signals, f_o, and the void fraction of the fluidized bed, ϵ, at various viscosities of the fluidizing medium.

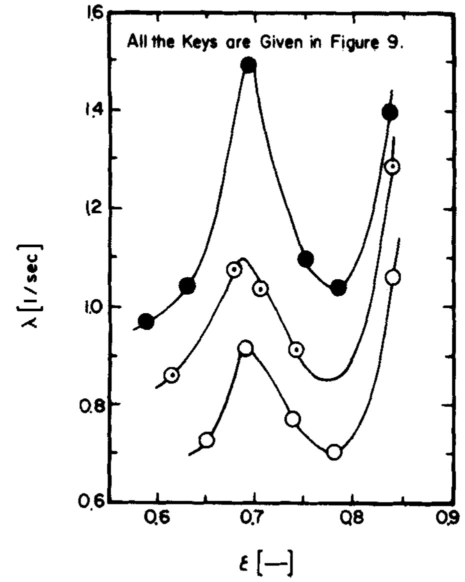

Figure 10. Relationship between the intensity of transition of particles, λ, and the void fraction, ε, at various viscosities of the fluidizing medium.

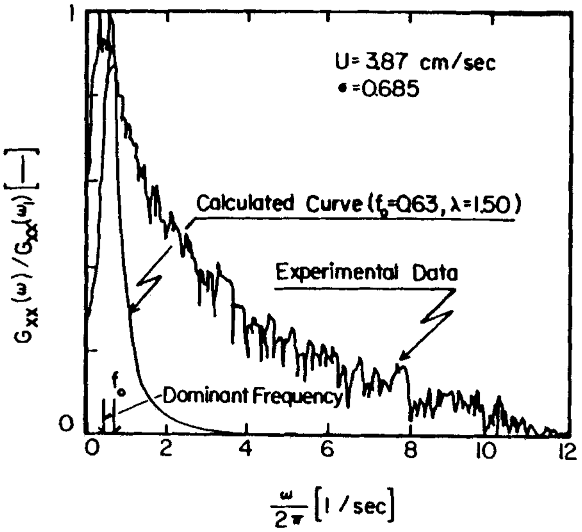

Figure 12. Comparison of the power spectral density function based on the model with that obtained from the experimental data.

ticle-liquid system, T_m the temperature of the medium or liquid, and ζ the friction factor of the shear forces exerted on the particle. Comparison of Eq. 20 with Eq. 21 indicates that the parameters, f_o, λ and $g(\mu)$, correspond to D, ζ and $f(T_m)$, respectively. Note that f_o is the frequency of particle movement in the fluidized bed, while the particle diffusivity, D, is proportional to the frequency of particle movement in the suspended Brownian particle-liquid system. The trend of variation of f_o as a function of ϵ exhibited in Figure 9 is similar to that of variation of D as a function of ϵ reported by Kramers et al. (1962). The decay constant of the auto-correlation function with respect to fluctuating forces in a stochastic process corresponds to the friction factor, ζ (e.g., Kubo, 1957); λ in the present model or Eq. 20 is essentially a decay constant. While the analogy between the "Einstein Relation," Eqs. 21 and 20 can not be ascertained rigorously at this time, it appears to provide information useful for analyzing the particle motion in a liquid-solids fluidized system.

Power Spectrum

The power spectral density function, $G_{xx}(\omega)$, is obtained from the auto-correlation function $R_{xx}(\tau)$ according to Eq. 14. Because of Eq. 8, the dominant frequency, $(\omega_1/2\pi)$, of the recorded signals, $X(t)$, must be approximately equal to the frequency, f_o, if the wave-like component of the signals is experimentally determined.

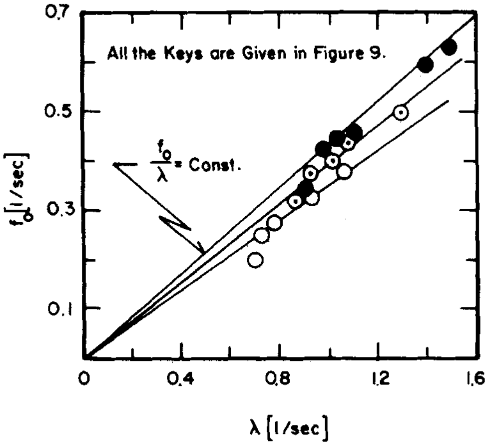

Figure 11. Linear relationship between the frequency, f_o, and the intensity, λ, for various viscosities of the fluidizing medium.

The experimentally determined function is compared with that calculated from the model, Eq. 15, in Figure 12. Notice that the value of $(\omega_1/2\pi)$ calculated from Eq. 19 is in good agreement with that of the frequency, f_o, determined from experimental data.

The power spectrum provides more detailed information than the auto-correlation function on the particle movement. Each frequency in the power spectrum corresponds to the velocity of a particle in the bed. It seems possible that the power spectrum approaches the particle velocity distribution if the size of the micro-capacitance probe is made sufficiently small.

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APPENDIX DERIVATION OF EQS. 6 AND 11

According to the assumptions expressed in Eqs. 3 and 4, we can write the probability balance for $p_A(t)$ as

$$p_A(t+\Delta t) = p_A(t)[1-\beta\Delta t] + [1-p_A(t)]\alpha\Delta t + o(\Delta t) \quad \text{(A-1)}$$

Rearranging this equation to form difference quotients and taking the limit as $\Delta t \rightarrow 0$, we obtain

$$\frac{d}{dt}p_A(t) + (\alpha+\beta)p_A(t) = \alpha \quad \text{(A-2)}$$

Solving this expression yields

$$p_A(t) = \frac{\alpha}{(\alpha+\beta)} + \left[p_A(0) - \frac{\alpha}{(\alpha+\beta)}\right]e^{-(\alpha+\beta)t} \quad \text{(A-3)}$$

which is Eq. 6 in the text. If

$$p_A(0) = \frac{\alpha}{(\alpha+\beta)},$$

the process is said to be second-order stationary (e.g., Chatfield, 1975).

The auto-correlation function of $Z(t)$ is defined as

$$R_{zz}(\tau) = E[Z(t)Z(t+\tau)] \quad \text{(A-4)}$$

We can see immediately that the product of $Z(t)$ and $Z(t + \tau)$ must take on only the values, 0 and 1. It will be equal to 1 if a particle is in the field at both times t and $(t + \tau)$; otherwise, it will be zero. Therefore, the expectation of the product is equal to the probability of this event:

$$E[Z(t)Z(t + \tau)] = Pr[\text{both } Z(t) \text{ and } Z(t + \tau) \text{ are } 1]$$
$$= Pr[Z(t) = 1]Pr[Z(t + \tau) = 1|Z(t) = 1] \quad \text{(A-5)}$$

Since $Z(t)$ is a time stationary process, and considering Eq. A-3,

$$Pr[Z(t + \tau) = 1|Z(t) = 1]$$
$$= Pr[Z(\tau) = 1|Z(0) = 1]$$
$$= \frac{\alpha}{(\alpha + \beta)} + \left[1 - \frac{\alpha}{(\alpha + \beta)}\right] e^{-(\alpha+\beta)\tau}$$
$$= \frac{\alpha}{(\alpha + \beta)} + \frac{\beta}{(\alpha + \beta)} e^{-(\alpha+\beta)\tau} \quad \text{(A-6)}$$

Substituting Eq. A-6 into Eq. A-5, we have

$$R_{zz}(\tau) = p_A(0)\left[\frac{\alpha}{(\alpha + \beta)} + \frac{\beta}{(\alpha + \beta)} e^{-(\alpha+\beta)\tau}\right]$$
$$= \frac{\alpha}{(\alpha + \beta)}\left[\frac{\alpha}{(\alpha + \beta)} + \frac{\beta}{(\alpha + \beta)} e^{-(\alpha+\beta)\tau}\right]$$
$$= \frac{\alpha^2}{(\alpha + \beta)^2} + \frac{\alpha\beta}{(\alpha + \beta)^2} e^{-(\alpha+\beta)\tau} \quad \text{(A-7)}$$

For the steady state operation, the probability that a particle will enter the field is identical to that for the reverse. Since the volume of the field compared with the total volume of the fluidized bed is very small, the intensity of exit from the field, β, is much larger than the intensity of transition from the outside to the inside of the field, α. For a small value of τ, the first term on the right-hand side in Eq. A-7 is negligible. Thus, Eq. A-7 may be rewritten as

$$R_{zz}(\tau) \simeq \frac{\alpha\beta}{(\alpha + \beta)^2} e^{-(\alpha+\beta)\tau} = \gamma e^{-\lambda\tau} \quad \text{(A-8)}$$

where

$$\lambda = (\alpha + \beta)$$
$$\gamma = \frac{\alpha\beta}{(\alpha + \beta)^2}$$

This is Eq. 11 in the text.

NOTATION

A	= amplitude of the sine wave component
c	= constant = $2\pi f_o$
D	= particle diffusivity
f_o	= cyclic frequency of the sine wave component
$G_{xx}(\omega)$	= power spectral density function of $X(t)$
$Pr[E_1\|E_2]$	= conditional probability of event E_1 given event E_2
$p(\theta)$	= probability density function of r
$p_A(t)$	= $Pr[Z(t) = 1]$
$p(x_1x_2;\tau)$	= joint probability density function of $X(t)$ and $X(t + \tau)$
$R_{xx}(\tau)$	= auto-correlation function of $X(t)$
T	= duration of the sample signal
T_m	= absolute temperature of the medium or liquid
U	= superficial velocity of the fluidizing medium
$X(t)$	= signal recorded from the capacitance probe
$Y(t)$	= sine wave component or wave-like signal
$Z(t)$	= stochastic or random component

Greek Letters

α	= intensity of transition of a particle from the outside of the field to the inside
β	= intensity of transition of a particle from the inside of the field to the outside
γ	= constant = $\alpha\beta/(\alpha + \beta)^2$
ϵ	= void fraction of the fluidized bed
θ	= initial phase angle with respect to the time origin
ζ	= friction factor of the shear forces exerted on the particle
λ	= constant = $(\alpha + \beta)$
μ	= viscosity of the fluidizing medium
ω_1	= stationary point of $G_{xx}(\omega)$

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